# Some Methodological Aspects of Particle Physics

## 1. Introduction

## 2. History of theoretical Developments in Elementary Particle Physics

2.1 Relativistic quantum Field Theory

2.1.1 Two Ingredients of RQFT

.2 Two Roads to RQFT

3. Fock Formalism

4. Reformulation of Stretching

2.2 Renormalization

2.2.1 Divergences in QFT

.2. classical Renormalization

3. Role of Lorentz Invariance for unambiguous subtraction of infinite quantities

2.3. Feynman Diagrams

2.3.1 Feynman's space-time formulation of QED

2. Closing the Computation Gap

3. Lamb shift & anomalous magnetic moment of the electron

4. The Nature of the divergences in Perturbation theory

2.4. The Analytic S-Matrix

2.4.1. Heisenberg introduces the S-Matrix theory

2. Non-Perturbative calculations of the S-Matrix.

3. Dispersion Relations

4. Singularities from unitarity & crossing

5. Analyticity & causality

6. The Mandelstam Conjecture

7. The Chew-Mandelstam heuristic strategy.

8. C.D.D. Ambiguities

9. The Chew-Frautschi Bootstrap

3. The Ultimate Nature of Matter

3.1. The Bootstrap picture

2. Tholeon Fundamentalism

3. Anaxicrondercion Fundamentalism

4. Mathematical Atomism

## 4 Conclusions

4.1. The structure of Scientific Theories
2. Correspondence relations
3. The role of surplus structure
4. The Floating model
5. The failure of Scientific Method

# Some Philosophical Aspects of Particle Physics

## 1. Introduction

1. Why should philosophers of science be encouraged to take an interest in particle physics?

In view of:- (a) Subject abounds in technical jargon, names & classification of several hundred particles

(b) characteristically theories in particle physics require rather elaborate mathematical development before any "practical" calculations can be carried out

(c) the literature of the subject is very extensive (amount published since 1930 exceeds publications prior to 1930 in all branches of physics)

2. Relevance of particle physics to philosophy of science.

(a) Ept is a modern, "live" branch of physics. Philosophy of science often deals with examples which are no longer of current interest in science Galileo & Newton or even non-relativistic quantum mechanics. For this reason physicists tend to feel philosophy of science is somewhat irrelevant – particularly since the character of theoretical physics appears to be significantly different from what it was in many of the historical examples. The study of ept gives an excellent opportunity of examining the truth of this claim.

(b) QFT is in a state of Kuhnian "crisis". Such paradigms as exist are generally regarded as inadequate. If one regards "normal" science as philosophically rather dull, following Popper, then the study of a "crisis" situation as it happens should be of considerable interest to philosophers of science.

(c) QFT thus provides an ideal testing ground for theories of how science develops. We can look at:—

(1) Methodologies of heuristics, how successive theories are, as a matter of historical fact, logically related to one another i.e. correspondence relations between theories. There is also the normative/prescriptive sense of specifying rules of discovery or heuristic strategems, which may themselves be derived from the descriptive historical analysis

(2) Methodologies of appraisal, how we appraise a theory once it has, for whatever reason and by whatever heuristic process, been proposed. This appraisal typically involves considerations of

(1) simplicity, single unifying theme

(2) empirical content, falsifiability

(3) empirical truth content, the degree to which a theory correctly predicts a large number of facts.

8 Novel predictions of S-matrix theory include (1) forward scattering dispersion relations

(2) Regge trajectory's for classifying particles and understanding their properties.

(3) understanding of inclusive reactions (Mueller)

(c) The degree to which we believe the theory to be true — this is connected particularly with the idea of successful novel predictions

Appraisal will involve discovering relation of theory to experiment which involves the following points

(1) Novel predictions which guide* experimental discoveries such as antiproton, $K^0$ regeneration, $\Omega^-$ and neutrino.
(2) Novel predictions which are verified "accidentally" such as the positron and Yukawa's meson.
(3) Crucial experiments whose success has given great impetus to a theory as e.g. Lamb-shift or $\Omega^-$.

(4) The Computation Gap which may ~~make a th~~ "insulate" a theory from experimental tests.

(d) In a reductionist programme ept is to be seen as a foundation for the whole hierarchical structure e.g. biology → chemistry → physics → ept.
But if ~~the~~ this foundation is itself "shaky" does this throw doubts on the whole programme.
Also the reductionist programme depends essentially on explaining

complex phenomena in terms of simple phenomena, but there are indications in the bootstrap philosophy that the putative simplest objects involve for their understanding consideration of complex objects. So in a sense the reductionist programme may be seen as circular thus
complex → simple → complex.

Another possibility is an open-ended infinite regress in which every elementary particle is itself resolved by more delicate probing into further constituents

At all events the idea of having with elementary particle physics reached a stable bed rock foundation for a reductionist programme seems to be illusory

(e) This leads us to pose the question: what light does it throw on the ultimate nature of matter? Is the atomistic programme still a valid one? Or should we subscribe to a bootstrap philosophy, or to Heisenberg's view of a unified field as a candidate for an Anaximandrian fundamentalism?

(F) Topics not discussed

(1) Role of symmetry in elementary Particle Theory. Ever since decisive role of symmetry constraints in devising new theories was emphasized by ~~Einst~~ Einstein in the development of Special Relativity in 1905, the same idea of deriving theories from symmetries instead of symmetries from theories has played a major role in particle physics. Examples are charge independence as expressed in the isotopic spin formalism of Heisenberg (1932), the introduction of the 'strangeness' quantum number by Gell-Mann and Nishijima (1953-1955), the discovery of non-conservation of parity in weak interactions suggested by Lee & Yang in 1956, the SU(3) symmetry classification of hadrons by Gell-Mann & Ne'eman in 1961, and the extension to SU(6) by Gürsey & Radicati and by Sakita in 1964.

But we can also regard symmetry as interesting properties derived from fundamental laws. We shall be concerned with attempts at spelling out the detailed dynamical laws, although in practice these always involves taking certain symmetry considerations into account.

P.T.O.

At all events the subject of symmetry has been treated in a separate paper "Symmetry in Intertheory Relations" (1975)

(2) Basic philosophical problems associated with quantum mechanics such as the theory of measurement, and how these problems look in the light of developments we shall be discussing which have taken place within the general context of quantum-mechanical ideas.
For our purposes we shall adopt a naive "fluctuation" view of quantal aspects i.e. observables are subject to quantum fluctuations. This view is not tenable in any simple sense but it will suffice for our purposes.

(3) Ontological status of elementary particles. We shall not discuss directly the question of whether the ~~sce~~ "ultimate" view of matter provided by QFT is more "real" than the familiar objects of sense experience (cf Eddington's two tables). We shall actually assume a realist approach to the interpretation of physical theories with due remarks on the notion of surplus mathematical structure

* name photon due to G.N. Lewis (1926)

# 2. History of theoretical developments in elementary Particle Physics

We divide the history of ept into four main divisions :—

| | | |
|---|---|---|
| Dirac | 1927 | Relativistic Quantum Field Theory |
| Bethe | 1947 | Renormalization → Feynman diagram techniques. |
| Mandelstam | 1958. | The Analytic S-Matrix → Bootstrap hypothesis |
| Weinberg-Salam | 1967 | Revival of field theories → gauge theories, non-local theories (strings and bags) |

---

Around 1927 the candidates for the elementary particles were the electron (discovered in 1897), the proton (1913, Moseley's study of X-ray spectra) and the photon (Einstein, 1905)*. Quantum Mechanics (Heisenberg 1925) and Wave mechanics (Schrodinger 1926) were really attempts to provide a theory of the electron (and less interestingly the proton).

but our story of the history of characteristically elementary particle theories will start with attempts to incorporate the photon in the new theoretical framework.
The reason why the photon is more typical than the electron or the proton of the many particles later to be discovered is the fact that photons can be produced (i.e. emitted) and annihilated (i.e. absorbed) so in general we want a theory that allows for a variable number of particles.
Now most of the elementary particles are spontaneously unstable, i.e. they disappear or decay after a very short time (even without interacting with an "observer") so clearly a theory which can deal with the photon is likely to be able to accommodate a description of the essentially evanescent character of elementary particles.
The reason why this aspect of the theory does not impinge on ordinary atomic and molecular physics is that the electron and proton are stable against spontaneous decay.

The appropriate methods for describing the appearance and disappearance of particles turned out to be relativistic quantum field theory.

# (1) Relativistic Quantum Field Theory

## 1. Two ingredients in RQFT

This is the application of ideas from relativity and quantum mechanics to the dynamics of fields i.e. systems with infinitely many degrees of freedom.

Relativity had demonstrated equivalence of mass and energy expressed as $E = mc^2$. This suggests that even for a particle at rest the rest mass $m_0$ might be interconvertible with energy of amount $m_0c^2$. So in a relativistic theory we expect possibility that particles of rest mass $m_0$ can be created by suitable input of energy with a threshold $m_0c^2$. Similarly annihilation of a particle may be possible with release of this amount of energy.

Quantum theory now allows for energy fluctuations $\Delta E$ in system related to life time $\Delta t$ of the quantum state of the system by the Uncertainty relation $\Delta E \Delta t \sim h$. So creation of a particle is possible provided particles annihilate with a time of order $\hbar/m_0c^2$ (i.e. after travelling a distance of $\sim \hbar/m_0c$). Particles produced in this way by spontaneous quantum fluctuations are called virtual particles.

Consequence is that in RQFT every problem (even the $N_0$-body problem i.e. the vacuum) becomes a many body problem.

## 2. ~~Here are~~ Three routes to RQFT formulation

~~(a) Field quantization~~

Debye ~~Ehrenfest~~ (1910) following suggestion of Ehrenfest ~~Debye~~ (1908)

~~suggested~~ derived Planck's radiation law by quantizing motion of the oscillators represented by the normal modes of the radiation field itself. Heisenberg's matrix mechanics was immediately applied to the same problem by Jordan (Born & Jordan (1925) Heisenberg, Born & Jordan (1926)).

A quite different approach was to start with the particle concept and derive Planck's law by regarding radiation as a gas of photons subject to the quantum statistics. This was the approach of Bose (1924). But neither Bose nor Debye (and later Jordan) discussed the detailed dynamical process involved in radiation and absorption. ~~The conceptual had been provided~~ The framework for a detailed approach had been laid by Einstein in 1917 with his theory of spontaneous and induced radiation probabilities, the so called A and B coefficients, but it remained for Dirac in 1927 to clarify 1) the relationship between the Debye and Bose approaches, 2) to actually provide a theoretical basis for calculating the Einstein coefficients. In particular what was the perturbing influence that caused spontaneous emission (the A coefficient)?

Dirac showed that we could treat the radiation field in two distinct ways and arrive at the same final result: a quantized field, which emitted and transcended

II Note ~~field~~ quantized field in second quantization refers to 1-particle S. Eq. — Hence term second-quantization. This field is a complex S-field unlike the real e-m. field which is subjected to quantization in the method of field-quantization

the wave and particle pictures of light (cf Bohr's complementarity view that the wave and particle pictures are alternatives applicable in complementary situations)

## Field quantization

classical field $\xrightarrow{\text{Field quantization}}$ Quantized Field

## Second Quantization

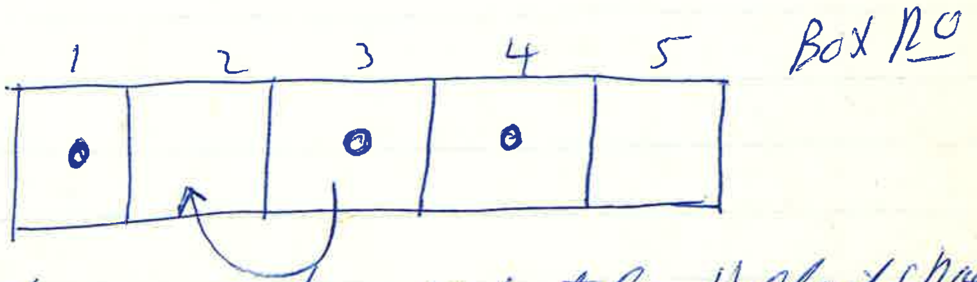
N classical particles $\xrightarrow{1^{st}\text{ quantization}}$ N-particle Schrödinger eqn $\xrightarrow{2^{nd}\text{ quantization}}$ Quantized Field.

\#

## 3 - Fock formalism

We explore the significance of second quantization using the ideas of Fock (1932) although the method is implicit in Dirac's 1927 paper.

We represent an N-particle state by locating each particle in a particular one-particle state

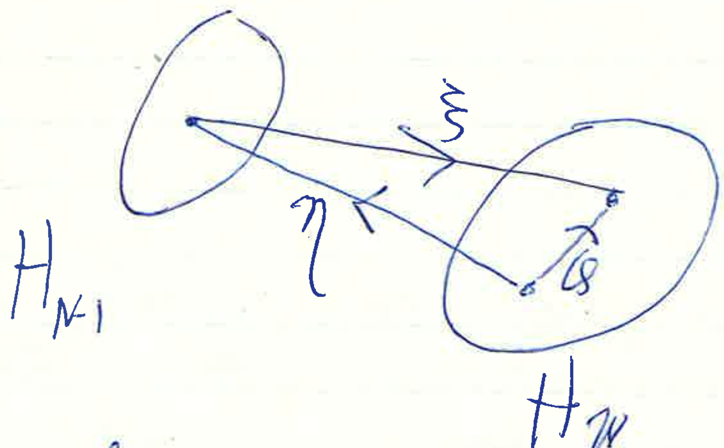



An operator in the N-particle Hilbert space $H_N$ acts by switching particles from one state to another e.g. particle in box 3 → box 2. But this can be thought of as a two stage process

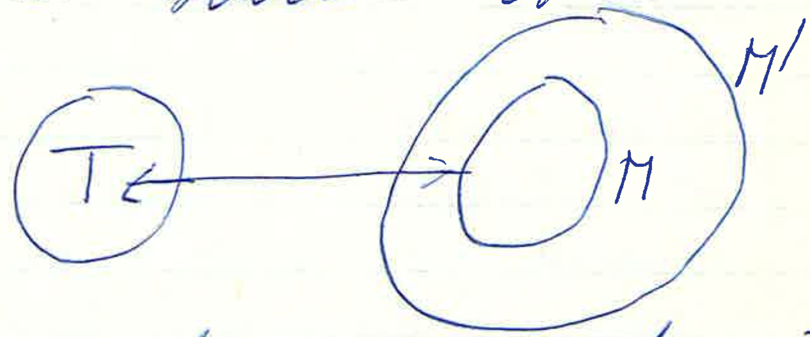
Particle comes out of box 3 leaving 2 particles only
then Particle is put into box 2 so we ~~can~~ again have 3 particles in all.

Schematically we introduce an operator $\eta$ which takes a particle out of a box and an operator $\xi$ which puts it back in (in general) another box.

So we "factorize" the whole operator denoted by $Q$ as $Q = \xi\eta$




So we have introduced $H_{N-1}$ as an element of surplus structure in our theory of $H_N$.




In general our theory now deals with a Fock space $H = H_0 + H_1 + H_2 + H_n$

But so long as we restrict ourselves to operators like $\xi\eta$ the Fock space is a mere mathematical device. But now the formalism can be extended very easily to describe creation & annihilation

Note: Reformulation involves a change in surplus structure
stretching usually involves ~~taking~~ ~~some~~ giving ontological reference to some of the new surplus structure (i.e. a realistic interpretation) cf Zahar (1973)

Note: paradigm shifts may involve a correspondence relation but ontology may ~~be~~ is changed — compare Watkins (1975) notion of revolutionary reduction. (I prefer to say revolutionary correspondence)

The correspondence relation involved in stretching is a case of radical reduction in Watkin's terminology (or radical correspondence)

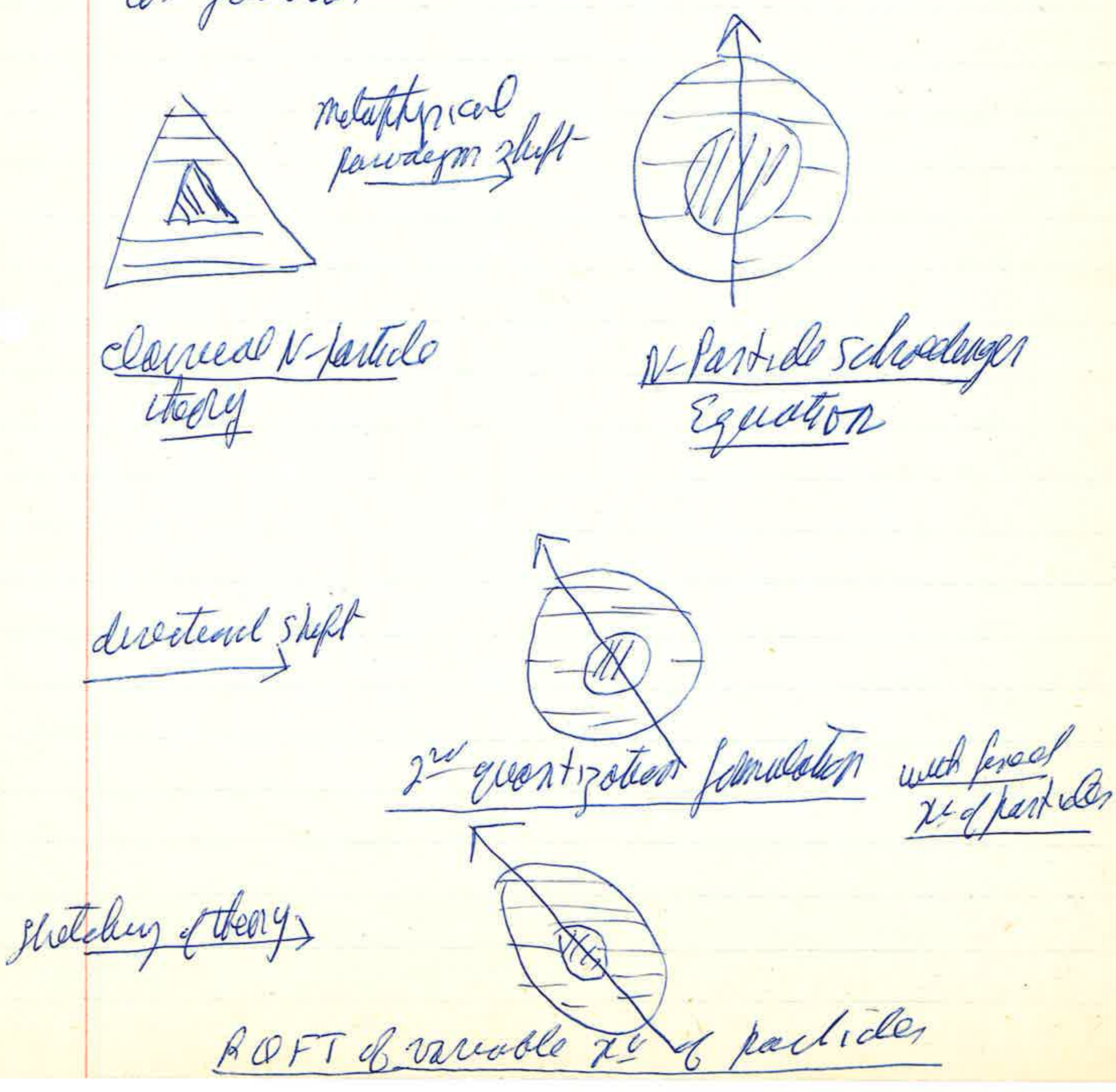
Note Watkins includes in reduction the case where theoretical structure (or content) of old theory is by passed

of particles. For example we might introduce an operator $\xi + \eta$ which would admit both processes.

Such linear combinations of "square root" operators are known as quantized fields and can also be introduced by directly "quantizing" the field amplitudes (electromagnetic potentials) which is the second method of approach.

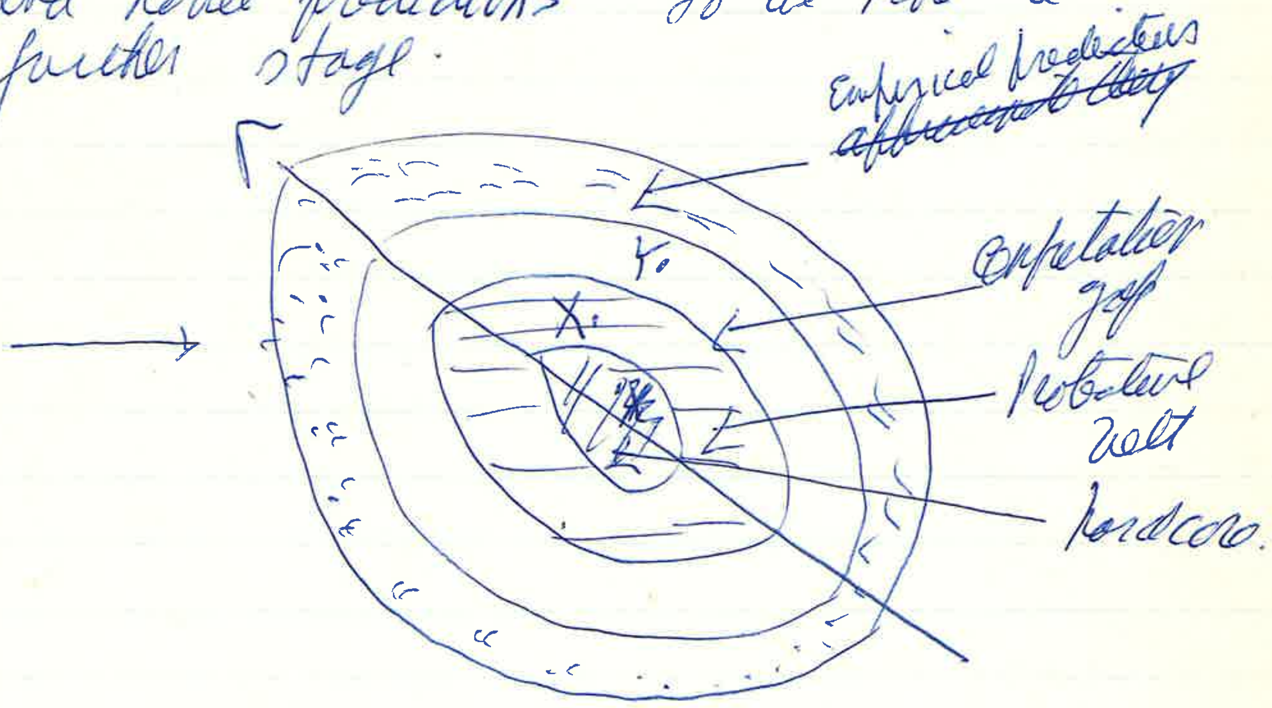
4. Reformulation and Stretching

We can represent the heuristic strategem involved as follows:



(Compare development of wave mechanics via Hamiltonian formulation of classical mechanics in terms of his characteristic function.)

But any new theory must be calculated with to produce testable consequences and novel predictions. So we have a further stage.




We can produce an approximate theory in two ways.

(1) alteration at X to produce soluble model i.e. for model computation gap is eliminated, such a model we refer to as model$_1$

or (2) alteration at Y to produce a ~~different~~ scheme of approximation this is also often referred to as working with a model which we designated as model$_2$

Model$_1$ can be regarded as a special case of model$_2$ in which change at X is regarded as being "moved into" the computation gap, but in practice distinction between model$_1$ and model$_2$ is usually clear.

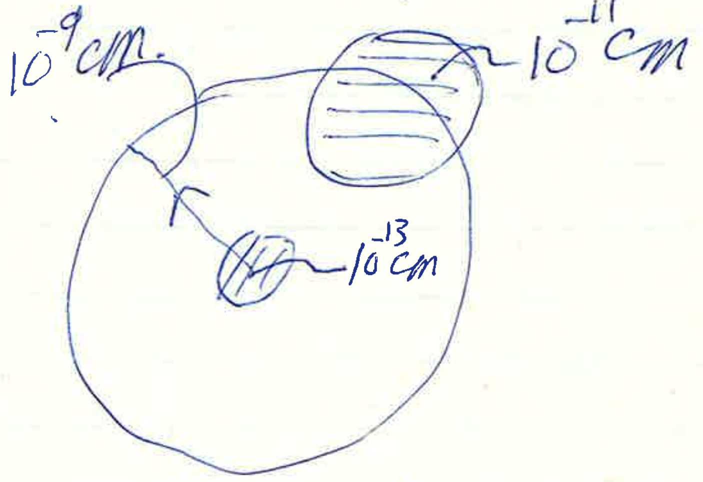
# (2) Renormalization

## 1. Divergences in Quantum field theory.

Dirac's theory was in a sense still-born. Ehrenfest pointed out that divergences would arise if the theory was used to calculate radiative reaction effects as occurs already in the classical theory with point electrons. In 1930 Waller worked out the self-energy of the electron and found a quadratic divergence, so the situation is actually more serious in quantum theory than in classical theory where the self-energy is linearly divergent. Indeed if one takes the theory seriously and calculates any quantity beyond the first non-vanishing order of perturbation theory one obtains infinite results. The situation is rather like set theory and the paradoxes. One can use "naive" set theory while knowing that the whole theory is actually inconsistent. The Patching-up operation of Russell's theory of Types can be compared with the Renormalization programme for dealing with the divergences.

The extra complication introduced by quantum theory is the effect of forced oscillations of the electron under the influence of the vacuum fluctuations of the field. This contributes to the self-energy of the electron over and above the classical effect arising from the interaction of the electron with its own electric & magnetic field. The self-energy problem is ameliorated but not eliminated in hole theory, the

self-energy divergence being only logarithmic (Weisskopf (1934)). The effect of hole theory is to "smear" the charge distribution of the electron over a distance of order $\hbar/mc$ due to ~~fault~~ the onset of Pauli repulsion on the virtual pairs produced by vacuum fluctuations in charge and current density of the electron field. The vacuum field fluctuations now interact with this extended charge distribution.




(This is a separate effect from the polarization of the vacuum by the electron's own electric field which produces an infinite effective charge, together with a finite effect modifying the Coulomb law between two charges (Uehling (1935)).

## 2. Classical renormalization

To deal with these infinities it may be suggested that the infinite contributions are absorbed into the definition of the mass and charge of the electron, the renormalized values being equated with the experimental values.

To see how this works in classical theory consider the equation of motion for an electron under the action of its own field. Lorentz showed we could write

$$m \ddot{\underset{\sim}{r}} = \underset{\sim}{F}^{(0)} + \underset{\sim}{F}^{(1)} + \cdots$$

where $\underset{\sim}{F}^{(0)} = -\cancel{2/3}\,\alpha \dfrac{e^2}{r_0 c^2} \ddot{\underset{\sim}{r}}$ , $\alpha \sim 1$

$$\underset{\sim}{F}^{(1)} = 2/3 \frac{e^2}{c^3} \dddot{\underset{\sim}{r}} \quad \text{etc.}$$

$$\therefore \; m' \ddot{\underset{\sim}{r}} = \underset{\sim}{F}^{(1)} + O(r_0)$$

$$m' = m + 2/3\, e^2/r_0 c^2 .$$

We now identify $m'$ with experimental mass, and then let $r_0 \to 0$, when we have the finite equation

$$m' \ddot{\underset{\sim}{r}} = 2/3\, e^2/c^3\, \dddot{\underset{\sim}{r}}$$

This approach to divergences in classical electrodynamics was especially advocated by Kramers from 1938 onwards in a series of publications.

The application to removing the more subtle divergences of quantum theory was ~~suggested by Kramers~~ at his ~~ideas were~~ used by Bethe to provide an explanation for the Lamb shift in the hydrogen spectrum (Lamb-Retherford 1947)

## 3. Role of Lorentz invariance for unambiguous subtraction of infinite quantities

The problem now was to show that an unambiguous subtraction procedure could be defined in which infinite contributions from all orders of perturbation theory could be consistently absorbed in renormalization of mass and charge of the electron.

But in general subtraction of infinite quantities is entirely ambiguous. To obtain a unique result agreeing with what one would expect from a "finite" theory it was necessary to formulate whole subtraction procedure in a manifestly Lorentz covariant manner. To see how this helps in the subtraction problem consider as a simple example evaluating

$$I = \int_{-a}^{b} x\,dx = \frac{1}{2}(b^2 - a^2)$$

$\lim_{a,b\to\infty} I$ is quite ambiguous e.g. with $b = a \to \infty$ $I = 0$, but with $a = b - 1/b \to \infty$, $I = 1$ and so on. $I$ is only conditionally convergent. But for a finite theory integrand would behave "properly" at infinity and value of $I$ would then be zero

(cf e.g. $I = \int_{-a}^{b} x e^{-x^2} dx = \frac{1}{2}\left[e^{-a^2} - e^{-b^2}\right] \to 0$ as $a, b \to \infty$.

But we can enforce correct value $I = 0$ by specifying that that region of integration is symmetric with respect to transfer operation $x \to -x$ which enforces the correct value $I = 0$

\# quotation from Feynman

"By forsaking the Hamiltonian method, the wedding of relativity and quantum mechanics can be accomplished most naturally".

this is the kind of argument used to ~~the~~ resolve ambiguities in subtracting infinite quantities in the renormalization programme.

The ~~relativistically~~ separate manifestly covariant formulation of QED was first provided independently by Tomonaga and Schwinger but their approach was soon superseded by the ideas of Feynman (1949) with his space-time approach to Q.E.D.

(3) Feynman Diagrams

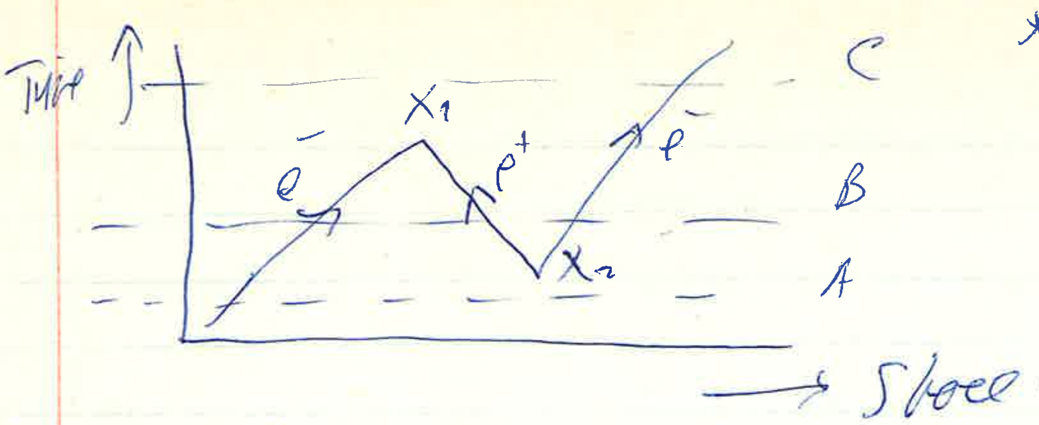
~~1~~ 1. Feynman's Space-time formulation of QED

This theory was derived from Feynman's reformulation of non-relativistic QM in terms of path-integrals (1948) and was extended to Q.E.D in 1949. The equivalence of Feynman's methods with the Schwinger-Tomonaga approach was demonstrated by Dyson (1949). Feynman contrasts his approach with the traditional Hamiltonian approach which considers a scattering process for example in terms of successive time-slices of the total space-time history of the particle.* Consider for example a process of pair creation and annihilation described in second order perturbation theory by the conventional formalism.

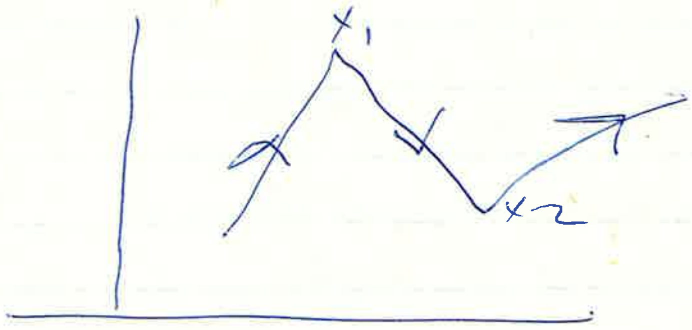
* quotation from Feynman

"It is as though a bombardier flying low over a road suddenly see three roads and it only when two of them come together and disappear again that he realizes that he has simply passed over a long switchback in a single road."

*

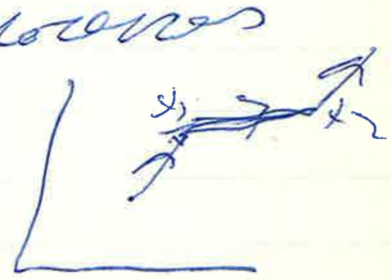



Consider 3 time slices at A, B & C.
At A there is one particle
B there are three particles
C there is one particle again
We describe this by saying a pair of particles is created at $x_2$ and one of the pair thus created annihilates the incoming particle at $x_1$.
Feynman draws the diagram thus:—




and says a single electron moves along a continuous trajectory in space-time — between $x_1$ and $x_2$ it propagates backwards in time with negative energy.
To obtain total effect of the scattering potential Feynman integrates over values for $x_1$, $x_2$, thus placing processes like


and




on an equal footing. The integrations being four-dimensional demonstrate the manifest Lorentz covariance of

* Compare Salam quotation
"An adequate notation is one which is both concise and intelligible to at least two persons, one of whom may be the author".

the formulation. These enable one to deal with the renormalization programme. ~~But also there is an enormous contribution towards closing the computation gap since processes which are apparently unrelated in old formalism are now all combined together in a single calculation.~~

## 2. Closing the Computation Gap

Another very important feature of the Feynman formulation is the enormous contribution it makes to closing the computation gap since processes which are apparently unrelated in the old formalism are now all combined together in a single calculation.

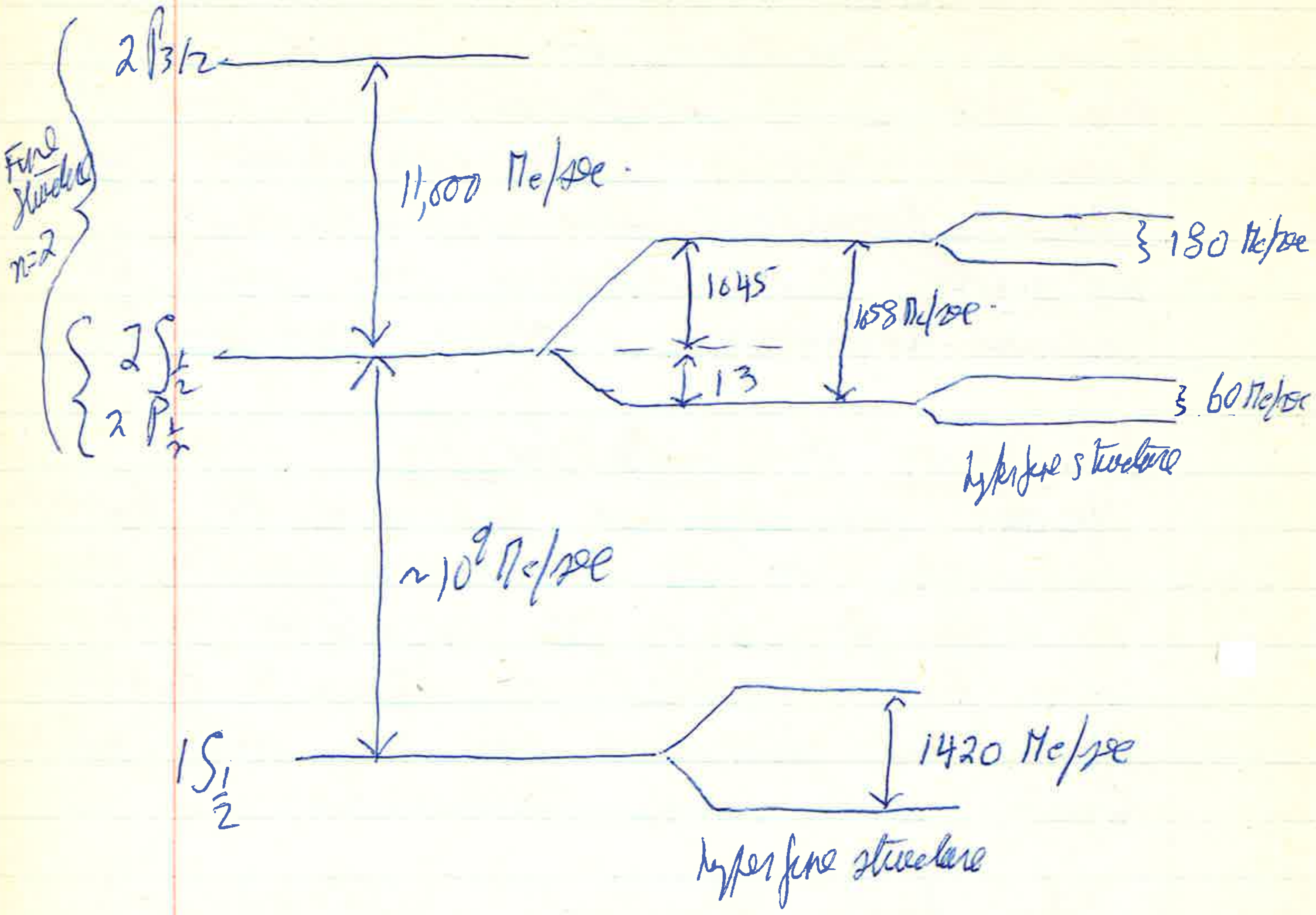
This has (1) a theoretical advantage:

It enabled Dyson (1949) to handle the very complicated proof of the renormalizability of Q.E.D to all orders of perturbation theory. (The gaps in the proof were filled in different ways by Ward & Salam in 1951) *

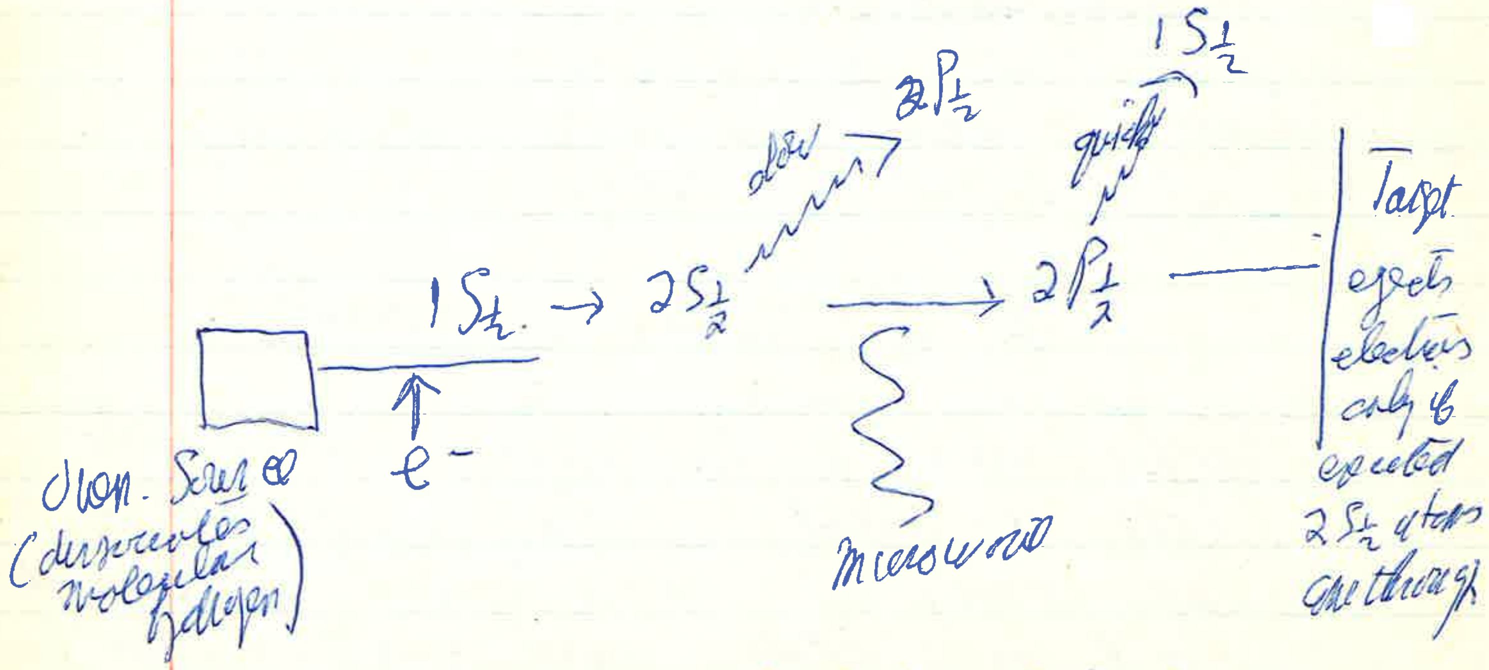
(2) a practical advantage

Higher order perturbation calculations (the so-called radiative corrections) can now be investigated by calculations which are still very complicated but not prohibitively so.

Thus Schwinger (1948) worked out the second order corrections to the




<u>Hydrogen spectrum</u>




<u>Lamb-Retherford experiment</u>

Coulomb scattering of an electron, while in 1952 Brown & Feynman obtained the fourth order corrections to the scattering of an photon by an electron (Compton scattering) and in 1953 Redhead solved the same problem for the scattering of an electron by an electron and of a positron by an electron.

14.3 The Lamb shift and the anomalous magnetic moment of the electron

But the most spectacular success of the new theory was the calculation of the Lamb-shift and the anomalous magnetic moment of the electron.

(1) Lamb shift

Spectroscopic evidence for anomalies in the fine structure of the hydrogen spectrum (doublet) date back to Houston (1926) — Pasternack (1938) interpreted anomalies in terms of an upward displacement of 2S level of about 1000 Mc/sec. (vacuum polarisation calculation of Uehling (1935) was of wrong sign and too small by a factor of ten to explain this shift)

separation in the Balmer lines $H_\alpha$: $n=3 \rightarrow n=2$

But Drinkwater, Richardson & Williams (1940) found no significant departure from the predictions of the Dirac theory.

Microwave measurements by Lamb & Retherford (originally suggested by Grotrian in 1928)

II Their result was revised for systematic error by Robinson (1963) and further revised by Robinson & Shyu (1970)

in 1947 were first to demonstrate and measure accurately the $2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$ shift in hydrogen. Their experiments were continued in period 1947–1953 and their final result was

$$\Delta E = 1057.77 \pm 0.10 \text{ Mc/sec.}$$

Experiment repeated by Robiscoe & Cosens (1966)[II] who also revised Lamb's analysis of his experiment (Robiscoe & Shyn (1970)) and the latest experimental result

$$\Delta E = 1057.88 \pm 0.06 \text{ Mc/sec.}$$

Theory developed by

(non-relativistic) Bethe (1947) → 1040 Mc/sec

(relativistic) Kroll & Lamb, French & Weisskopf (1949) → 1052 Mc/sec. (6 Mc too low)

Salpeter (1953) renormalized more accurate treatment Coulomb field + approximate 4th order corrections → 1057.2 Mc/sec (~½ Mc too low)

Using corrected 4th order magnetic moment and more accurate treatment Coulomb field Layzer and Fried & Yennie (1960) → 1057.70 ± 0.15 Mc/sec.

More accurate 4th order calculation by Soto (1960) led to value → 1057.56 Mc/sec

By 1970 experimental value showed real discrepancy, Applequist & Brodsky (1970) found mistake in Soto's 4th order calculation → 1057.91 ± 0.16 Mc/sec.

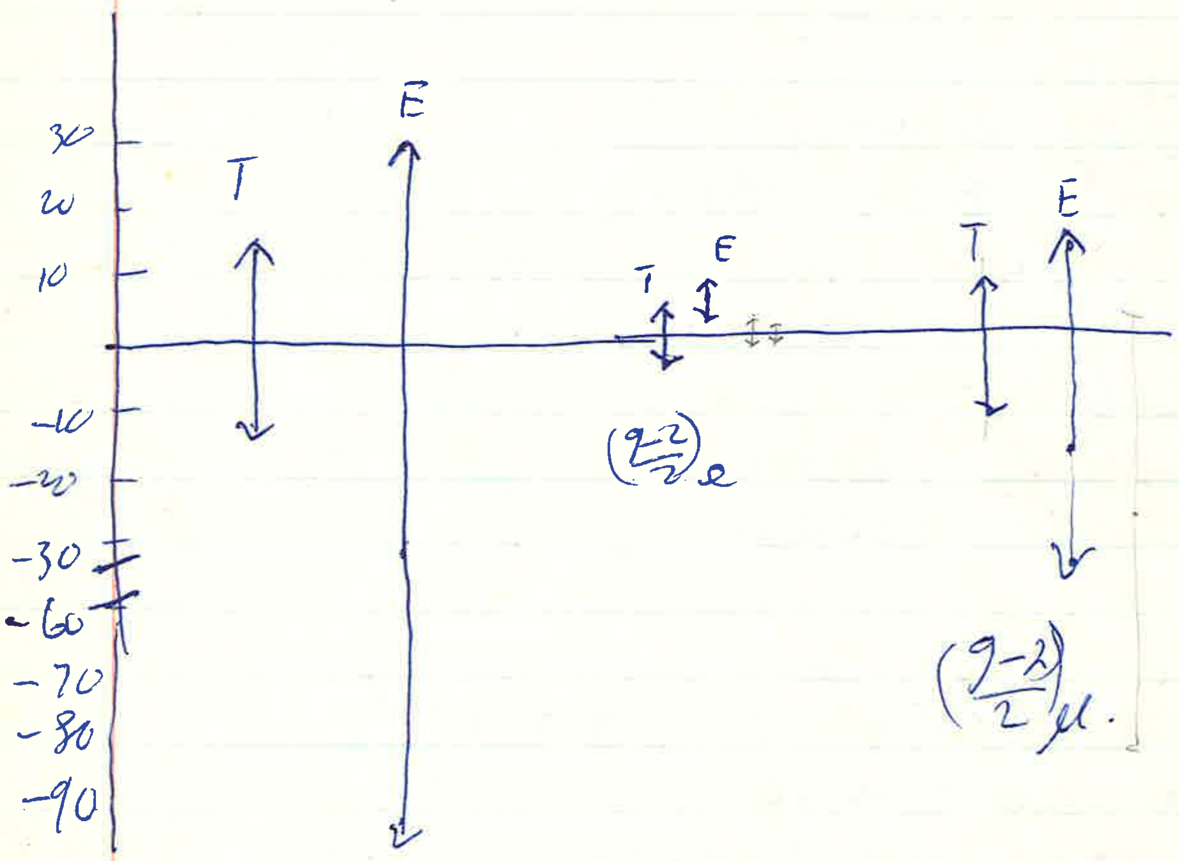
Latest value quoted in Lautrup, Petermann & de Rafael review (1972) is 1057.911 ± 0.012 Mc/sec

accuracy p.p.m

| | Theory | Expt | D(Expt-Theory) |
|---|---|---|---|
| Lamb | ± 12 | ± 60 | − 31 |
| $\left(\frac{g-2}{2}\right)_e$ | ± 2.2 (± 0.6) | ± 3.5 (± 0.2) | + 5 0 |
| $\left(\frac{g-2}{2}\right)_\mu$ | ± 10 (± 13) | ± 27 (± 27) | − 13 (− 26) |

(1977 values in brackets)

7th figure




Lamb

(2.) Anomalous magnetic moment of the electron

First deduced from measurement of hyperfine structure in hydrogen & deuterium by Nafe, Nelson and Rabi (1947).
Breit (1947) suggested discrepancy between theory & experiment in NNR experiment was due to ~~anomalous~~ an intrinsic magnetic moment for the electron.
Kusch & Foley (1947) and (1948) tested Breit's suggestion by measuring Zeeman splitting of levels in Gallium — confirmed Schwinger's (1948) calculation of anomalous moment due to radiative corrections.
By 1952 value of $\frac{g-2}{2}$ = 0.0011465 ± .000012 confirming Karplus & Kroll's calculation. Bohr magnetons ($e\hbar/2mc$)
(Koenig, Prodell & Kusch (1952))

But Franken & Liebes (1957) found a value 0.001165 (11), much too high for Karplus & Kroll.
This led to Sommerfield and Petermann's recalculation of Karplus & Kroll's result in 1957.
Meanwhile experiments on free electrons (using rotation of polarization of beam of electrons by Larmor precession in a magnetic field) was initiated by Louisell, Pidd & Crane in 1954, and refined by Schupp, Pidd & Crane in 1961 who found 0.0011609 (24). This experiment had errors same order as 4th order correction so new experiment undertaken by Wilkinson & Crane in 1963.

* Most accurate measurement is due to Dehmelt et al (1977) observing spin flips on a single electron trapped in a magnetic bottle.

value obtained for electron anomaly is

0.001159652.4 ± 0.2

Result of Wilkinson & Crane was .00 1159622 (27) in good agreement with theory.
But in 1968 Rich re-evaluated the W-C result and got a value .00 1159 549 (30) which was 3 standard deviations too low.
So experiment was repeated by Rich & Wesley in 1971. They obtained .00 1159657.7 ± 3.5 which could pick out the sixth order contributions to the anomaly (which are of order 11 ppm)
Granger & Ford (1972) re-evaluated RRW value to give 0.00 1159656.7 ± 3.5
They also corrected W & C result and brought it into line with the accurate W & R result (acknowledged in Wesley & Rich 1972 ROP review article).

*

Theory developed by

Schwinger (1948) .00 1161 Bethe's approx. to 2nd order
Karplus & Kroll (1950) .00 1145.
(1st 4th order calculation in Q.E.D)

corrected by Summerfield (1957), Petermann (1958), Kroll (unpublished) } .00 11596 (close to Schwinger's old value)

6th order corrections calculated by several groups.
Levine & Wright (1973)
.00 1159651.9 ± 2.5
correcting an earlier calculation of
.00 1159655 ± 2.

* Prof Farley reported on preliminary muon result in November 1975 at I.C. seminar
"How deeply is the muon to be understood via a [illegible] theory".

Latest theoretical value for $\left(\frac{g-2}{2}\right)_\mu$ is quoted by Calmet et al. (1977) as
$0.001165920.6 \pm 12.9$

Best theoretical value for $\left(\frac{g-2}{2}\right)_e$ is given by Calmet as
$0.001159652.4 \pm 0.6$

Probably most reliable ~~the~~ calculation is by Cvitanovic & Kinoshita (1974) who obtain .00 1159651.7 ± 2.2
(compared with expt .00 1159656.7 ± 3.5)

Note hadronic contributions would affect tenth significant figure

for the anomalous magnetic moment of Muon

latest measurement by Bailey et al (1975) gives $\left(\frac{g-2}{2}\right)_\mu$ = .00 1165895 ± 27

while theory is .001165908 ± 10
(includes 73 ± 10 from $a_\mu$ (hadrons))

*

This example supports the Zahar view that novel predictions need not be temporally novel and also stresses the importance of quantitative predictions in assessing theories

cp example of classical celestial mechanics and also calculations of Hylleraas and later ~~Pek~~ Pekeris and Kinoshita on the ground state of helium.

A Bayesian account of how quantitative predictions affect subjective probabilities of theories is given by Redhead in his paper "The Logic of Comparative theory evaluation".

†† A power series in $\lambda$ which is convergent for any value of $\lambda$ is absolutely convergent for any smaller of $\lambda$.

* note asymptotic expansion does not fix the corresponding function uniquely — Dyson shows extra assumptions may be needed for this.

4 The Nature of the divergences in perturbation theory

There are two quite separate questions

(1) are the renormalized quantities themselves finite (i.e. exist) and not just a divergent sum of finite terms?

The Answer is not known for a complicated theory like Q.E.D. but model calculations by Hurst (1953) & Thirring ~~and Hurst~~ (1953) & Hurst (1952) for $\lambda\phi^3$ scalar theory show renormalized series is not ~~absolutely~~ convergent A

( no of graphs of order $n$ is $\sim n^{n/2}$ and contribution of each graph is $\sim 1/n^2$ so series behaves like $\sum \lambda^n \cdot \frac{n^{n/2}}{n^2} \sim \sum \lambda^n n^{(n/2-2)}$ diverges for all $\lambda$ )

Defect in this argument is that the $n^{th}$ order term in this ~~[illegible]~~ series could be conditionally convergent to zero by cancellation of signs — very difficult to investigate — discussed by Riddell (1953)

Probably series is asymptotic

( i.e. $f(z) = \sum a_n z^n + b_n z^{-n-1} \sim \sum A_n z^n$ for limited region of $\arg z$.
and $\lim_{z \to 0} \frac{f(z) - \sum_0^n A_n z^n}{z^n} \to 0$. for all $n$.

so each partial sum approximates $f(z)$ "more closely than" $z^n$ as $z \to 0$. ) *

Typically error involved is smaller than last term calculated, but after a while

* Note recent work by Glimm & Jaffé (1969–1972) who solved (2+1) dimensional theories with $\phi^3$, $\phi^4$, $\bar{\psi}(x)\,\phi(x)\,\psi(x)$ *without* perturbation series. Infinities appear in the *exact* solutions just as in the perturbation theory so presumably it is not perturbation theory which is at fault.

this last term starts to increase, since series actually diverges as $n \to \infty$!

There are two arguments for asymptotic nature of expansion

(a) Dyson (1952) claims that if $e \to ie$ the series cannot be expected to converge or indeed to correspond to "a well-defined function" due to the fact that in such a world repulsion of opposite charges would lead to an "explosive disintegration of the vacuum by spontaneous polarization."

(b) Hurst (1952) argues "excellent agreement between experimental results and theoretical calculations would indicate that the series is in fact to be interpreted as an asymptotic expansion about the singular point $\lambda = 0$"

(2) Are the renormalization constants finite?

for exact solution of renormalized interacting fields: *

Källén (1953), Redmond (1958) argue renormalization constants may actually be finite and appear infinite because the relevant functions are not analytic ~~at $e = 0$~~

e.g. $e^{-e^2 L} \simeq 1 - e^2 L + \frac{e^4 L^2}{2} - - -$

as $L \to \infty$ L.H.S. $\to 0$ but each term in perturbation expansion is apparently infinite.

considerable light on the "defects" of renormalization theory has been thrown by the work of Haag (1955).

Haag's theorem shows that under rather general restrictions sets of operators referring to free fields and to interacting fields cannot belong to equivalent representations of the canonical equal-time commutation relations i.e. cannot be connected by a unitary transformation.
This implies that Dyson's U operator for finite times between both the Heisenberg & interaction representations does not exist. As Barton (1963) puts it: "with this fact in mind the occurrence of formal divergences in the theory is to be expected and should in no way surprise one". Roman (1969) comments " We may now wonder why in spite of its non-existence, the interaction picture leads, at least in perturbation theory to reasonable results. The 'mystery' resembles, in a sense, the indiscriminate manipulations of ordinary point-quantum-mechanics when one often deals with not bounded operators and non-normalizable states, pretending that the structure from which one deduces sensible results exists. Of course, there always comes a point when we must realize the inadmissibility of the manipulations and the emergence of a nonsensical result then forces the physicist to adopt, eventually, a mathematically rigorous framework".
To circumvent Haag's theorem, we have to allow the dynamics to select the appropriate inequivalent representation

The existence of non-equivalent representations is associated with the fact that the dimensionality of the Hilbert space associated with a system with an infinite number of degrees of freedom is non-denumerable, i.e. the Hilbert space is non-separable.

In the late 1960's a new formulation of field theory was introduced by Schwinger with his method of sources (sorcery). Schwinger allows his actual fields to interact with an unquantized source field whose strength is ultimately allowed to go to zero. The source field is effectively used to "probe" the structure of the actual fields. Using methods of functional analysis S. has given a new formulation of renormalization theory which, he claims, avoids the ambiguities of the 'conventional' approach, and over and above providing a reformulation of conventional field theory, ~~is~~ ~~intended~~ leads to an 'approach to particle physics which S claims to be intermediate between and superior to either field theory and S-matrix theory. (cf Schwinger: Particles & Sources 1969) "A theory with field theory the physical emphasis upon space & time, but it is not an operator theory - Like S-matrix; it is phenomenological in its emphasis upon the actual physical system." S. sees his method as the dual of the current algebra approach ("currents are supposedly more physical than fields")

¥ ~~Kogut & Susskind (1973)~~

"~~It seems to us very unlikely that partons are really point particles.~~
... ~~a parton will be resolved~~ even ~~further into yet another hierarchy~~ of ~~constituents~~."

# (4) The analytic S-Matrix

## 1. Heisenberg introduces S-Matrix theory

The research programme of the analytic S-matrix derives from two strands (a) a thesis about what a fundamental theory of elementary particles should refer to viz the S-matrix, (b) a new non-perturbative method of calculating the S-matrix.

Heisenberg (1943) introduced the S-matrix or scattering matrix as the fundamental entity of interest by asking two questions (~~original~~ First to use S-matrix explicitly was Wheeler 1937 (1940?) in nuclear physics.)

(a) If a "complete" theory should involve a fundamental length what features of existing theories might be expected to survive in such a 'complete' theory (cf Einstein's attitude to relativity as having "survival value" beyond a suspect (due to photon effects) classical electro~~dynamics~~magnetism described by Maxwell's equations).

(b) Should not a theory restrict itself anyway to what can actually be observed (cf his original ~~into~~ intention for introducing matrix mechanics).

Heisenberg suggested answer to (a) and to (b) (~~in view of fundamental length~~) was the specification of the S-matrix

which would comprise two sorts of information

(a) scattering or reaction cross-sections derived from ~~scattering~~ transition amplitudes from an arbitrary initial to an arbitrary final state.

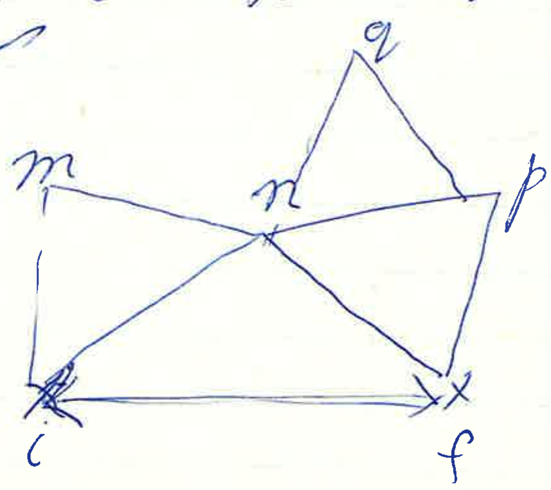
(b) Bound-states and resonances (short lived unstable particle states) would be related to singularities in the S-matrix, at possibly unphysical values of its arguments. The idea here is that anomalies or "bumps" in scattering cross-sections are connected with formation of unstable but relatively long-lived "complexes" composed of the incoming particles.

## 2. Non-Perturbative Calculations of the S-Matrix

The success of QED is due to the applicability of perturbation theory which is related to the small value of the fine structure constant ($e^2/\hbar c \sim 1/137$) i.e. to the weakness of the electromagnetic interaction. For hadron physics the particles interact via the strong interaction which invalidates the perturbative approach (contrast e.g. the atomic bomb with chemical high explosive) i.e. to apply field-theoretic approach to calculate of S-matrix required new schemes of approximation

For example we could consider a limited number of virtual particles in an arbitrary number of states (Tamm-Dancoff approximation) or an arbitrary number of virtual particles in a limited number of states (Tomonaga approximation). But no satisfactory method of dealing with renormalization could be found and no adequate account of $\pi N$ scattering was achieved. Success in accounting for features of low energy $\pi N$ scattering (the 3-3 resonance in the P-wave for example) resulted from the Chew-Low-Wick model which involved a quite different approach and involved expressing the scattering amplitude for a real (renormalized) physical process in terms of scattering amplitudes for all "real" processes ~~begin~~ which could ~~begin~~ connect with both the initial and final states.

We represent the situation schematically as follows




It was soon demonstrated (Oehme (1955)) that the Chew-Low model was an example of a dispersion relation and was connected with analyticity properties of the S-matrix.

## 3. Dispersion Relations

The dispersion relation approach to calculating the S-matrix involve the following sequence of ideas.

(a) We consider the S-matrix elements as function of energy (and other variables) and we now allow the energy to assume complex values.

(b) We assume S-matrix is an analytic function except for certain singularities (By Liouville's theorem a bounded analytic function with no singularities is necessarily a constant)

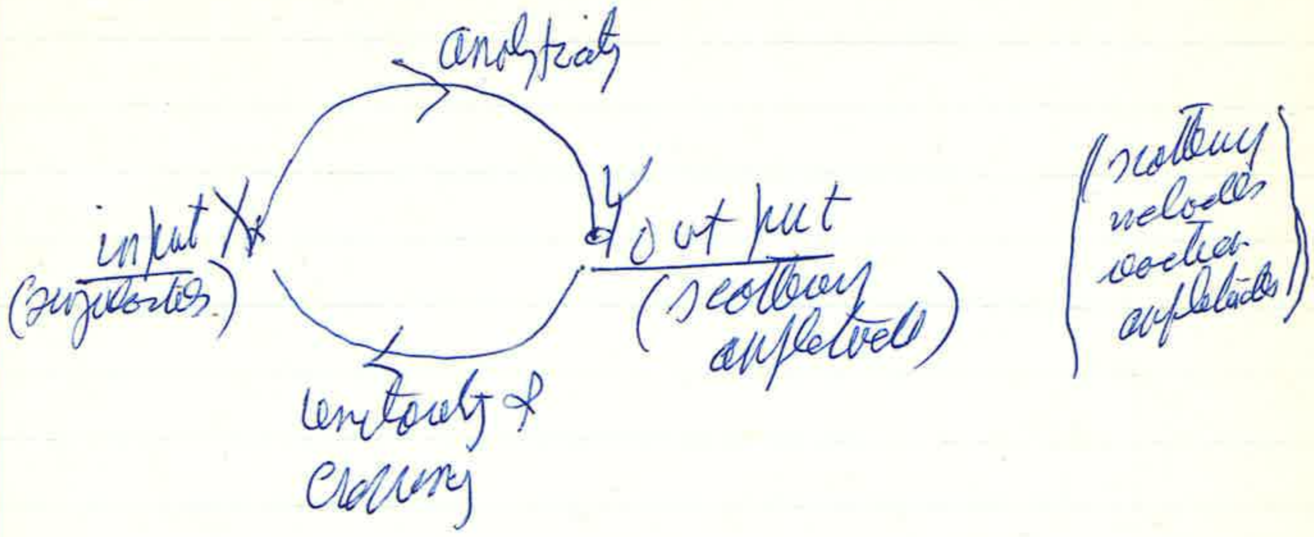
(c) We use Cauchy's theorem to relate scattering amplitudes to the singularity structure (position of singularities and behaviour of the function in the neighbourhood of the singularities e.g. residues at poles and discontinuities across branch cuts.)

(d) We use unitarity & crossing principle to locate part of the singularity structure (see below)

(e) We assume there are no other singularities than those demanded by unitarity & crossing (Principle of maximal analyticity of first kind) or the Mandelstam Conjecture

* (F) A Principle of Maximum strength is used to eliminate arbitrariness in strength of coupling constants.

(d) We now have a ~~coupled~~ feedback situation




e.g. we hypothesise analyticity $Y = X$
unitarity $X = Y^2$

Soln here is $Y = 0$ or $1$ i.e. non-unique

(e) We look for possible ambiguities in soln of equations and seek to remove them by a principle which eliminates certain singularities. This is achieved by principle of maximal analyticity of second kind which is expressed by considering angular momentum as well as linear momentum as a complex variable (can be expressed by "Every pole is a Regge pole, i.e. lies on a Regge trajectory in complex angular momentum plane)

In our analogue model we could write e.g. soln must be invariant under reflection in origin – would eliminate the $Y = 1$ soln and leave an unambiguous result $Y = 0$. *

## 4. Singularities from unitarity & crossing

We indicate briefly the steps which link unitarity, which enforces the conservation of probability, with the singularity structure

(a) Consider a function $f(z)$ which satisfies the Schwarz reflection principle

$$f(z^*) = f^*(z)$$

, scattering amplitudes satisfy a ~~this prin~~ principle of this type.

(b) Singularities in such a function ~~are~~ on the real axis are identified by the appearance of an imaginary part for $f(z)$ in the neighbourhood of the real axis

because if $f(x+i\varepsilon) = u + iv$

then $f(x-i\varepsilon) = u - iv$

So there is a discontinuity of $2iv$ in crossing the axis which indicates a branch cut, the "threshold" for which is a branch point.

Poles at $x=a$ poles are also identified by writing

$$\cancel{f(z) \sim \beta/z-a = \beta P\left(\frac{1}{z-a}\right)}$$

$$f(x \pm i\varepsilon) \sim \frac{\beta}{x \pm i\varepsilon - a} \quad \cancel{\pm i\pi\delta(} = \beta P\left(\frac{1}{x-a}\right) \mp i\pi\delta(x-a)\times\beta$$

So again the imaginary part identifies the position & residue of the pole.

(c) If we take for $f$ a scattering amplitude then $\operatorname{Im} f$ for forward scattering is connected with the absorption of particles from the incident beam (cf the imaginary part of a refractive index functioning as an absorption coefficient)

But by conservation of probability (and this is where unitarity comes in) absorption of particles from incident beam is related to rate at which particles are emitted into all possible reaction channels representing scattering or production processes.

This situation generalizes to the case of non-forward scattering and serves to identify branch points on the real axis with thresholds at which a new reaction channel becomes energetically possible. The case where the reaction channel involves the formation of a single particle leads to the "degenerate" case of δ-function for the imaginary part of the amplitude i.e. to a pole – referred to as a particle pole.

Reversing the above chain of argument it becomes plausible that we have the following chain of connections

Reaction amplitudes → Im. Scattering amplitude

→ singular structure of particle poles & normal threshold branch-points.

So here we have the sought-for connection between transition amplitudes and part of the singular structure.

In the same way crossing states that one scattering amplitude may be related to another by analytic continuation to

unphysical values of energy & momentum transfer (in simplest case of 2-particle elastic scattering) without new forces singularities in those crossed channels which appear ~~as~~ in the direct channel as singularities at unphysical values of the energy & momentum transfer arguments.

## 5. Analyticity & Causality

The question now is how do we know there are no other singularities than those enforced by covariance & crossing, e.g. singularities ~~in the~~ off the 'real' ones at complex values of the arguments.

The first approach to this problem is to try and link analyticity with causality after the fashion used in deriving dispersion relations in optics (Kramers-Kronig relations) which connects the real and imaginary parts of the refractive index, or similar results in the theory of electrical circuits etc.

These classical results depend on the following mathematical result.

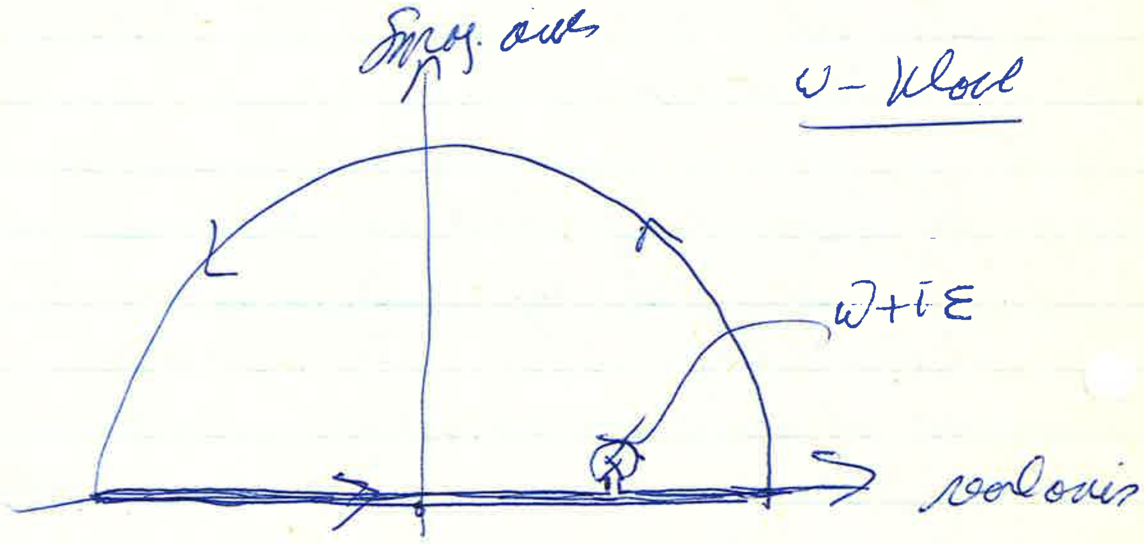
Consider some physical system which exhibits a linear causal response $H(t)$ to an input $G(t)$. Then the response function $L(\tau)$ where $H(t) = \int_{-\infty}^{\infty} L(t-t')\, G(t')\, dt'$.

* Theorem invoked here says.

If $k(z, \omega)$ is analytic function of $z$ for all values of $\omega$ on the path of integration then integral

$$f(z) = \int_{\Gamma} k(z, \omega)\, g(\omega)\, d\omega$$

is an analytic function of $z$ so long as the integral converges absolutely.
($g(\omega)$ need not itself be analytic)




II

the causality condition is $L(\tau) = 0$ for $\tau < 0$
We are interested in properties of the Fourier transform

$$\tilde{L}(\omega) = \int_{-\infty}^{\infty} L(\tau)\, e^{i\omega\tau}\, d\tau$$
$$= \int_{0}^{\infty} L(\tau)\, e^{i\omega\tau}\, d\tau$$

this is an analytic function of $\omega$ in the upper half-plane. (assuming $L(\tau)$ is polynomial bounded in the upper half plane)*

$\tilde{L}(\omega)$ is related to forward scattering amplitude for S.H. wave of frequency $\omega$ in optical applications and this in turn is related to the refractive index.
So $\mathrm{Re}\, \tilde{L}(\omega)$ relates to dispersion
$\mathrm{Im}\, \tilde{L}(\omega)$ relates to absorption

Dispersion relations connecting these two quantities can now be written down

$$\tilde{L}(\omega + i\varepsilon) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\tilde{L}(\omega')\, d\omega'}{\omega' - \omega - i\varepsilon}$$ #

where $\mathrm{Re}\, \tilde{L}(\omega) = \frac{1}{2\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, \tilde{L}(\omega')\, d\omega'}{\omega' - \omega} + \frac{1}{2}\, \mathrm{Re}\, \tilde{L}(\omega)$

$\therefore\ \mathrm{Re}\, \tilde{L}(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, \tilde{L}(\omega')\, d\omega'}{\omega' - \omega}$

$$= \frac{2}{\pi} P \int_{0}^{\infty} \frac{\mathrm{Im}\, \tilde{L}(\omega')\, \omega'\, d\omega'}{\omega'^2 - \omega^2}$$

if we have $\mathrm{Im}\, f(-\omega) = -\mathrm{Im}\, f(\omega)$ crossing condition which holds in the optical case.

To pursue this idea in QFT we can try to derive analyticity properties from microcausality as expressed in the canonical equal time commutation relations for observables

$$[O_1(x_1, t_1), O_2(x_2, t_2)] = 0$$

if $(x_1 t_1) \rightarrow (x_2 t_2)$ is space-like so two points cannot be connected by a light signal.

Difficulty is that we cannot rigorously derive analyticity results in sufficiently large domains to exploit the full dispersion relations approach. Some partial results can be obtained e.g. for $\pi$-N scattering fixed t dispersion relations are provable for $0 \leq -t \leq 18 m_\pi^2$?

We want and need complete information about analyticity properties in all variables (e.g. s and t for 2-body scattering)

## 6. The Mandelstam Conjecture

Another approach is to examine the analytic properties of particular Feynman graphs and try to "infer" analytic properties of the full amplitude.

Mandelstam investigated some 4th order Feynman diagrams and showed that a particular representation (as a double dispersion relation) was possible.

In 1958 he adopted a bold innovation. Instead of trying to derive analytical properties from field theory, he was guided by the successes of such dispersion relations that can be derived (e.g. forward scattering of π-N system) and also use what we know from enumeration of Feynman graphs and now make a conjecture as to what the analytic properties i.e. the singularity structure is for the full amplitude.

The Mandelstam Conjecture as formulated in his 1962 review article reads "The scattering amplitude is analytic ~~at~~ in all its variables except at those points where singularities arise as a consequence of the unitarity condition."

In his 1958 paper Mandelstam went on to assume that the singularities dictated by unitarity would permit a specific representation — the Mandelstam representation. The latter assumption, although very fruitful

* Also Landau advocated abandonment of field theory — these doubts of field theory date from the 1930 – cf Peierls & Landau

turned out not to be true in perturbation theory (Mandelstam, 1959). Nor was Mandelstam able to prove his representation to be generally valid from axiomatic field theory even for cases where perturbation theory suggested it might hold (Mandelstam N.C. 1960).
However the Mandelstam representation was proved for non-relativistic potential scattering (by Yukawa interactions) by Blankenbecler, Goldberger, Khuri and Treiman in 1960 and by Regge in 1959 using his method of complex angular momentum.

Mandelstam himself regarded his conjecture as just that a conjecture about how field things should behave. He did not subscribe to the use made of his ideas by Chew, Frautschi, Stapp & others. In his 1962 review he claims (1) S matrix theory contains less than can actually be measured
(2) analyticity assumptions as assumed "are rather artificial".
The idea that dispersion relations could actually replace field theory goes back to Gell-Mann (1956 Rochester Conference)
But Mandelstam's ideas were taken up enthusiastically by Chew, Frautschi, Stapp + etc and lead to the

* this idea can actually be traced back to Kroenig (1946)

# note how Fourier had enabled physics to deal with non-ok analytic functions — Chew now reverts to view that basically we should only consider analytic functions.

idea of an S-matrix theory quite independent of field theory in which the Mandelstam conjecture would now play the role of a postulate.*
The question whether this postulate could be derived from causal local field theory was considered irrelevant.

In the development of analytic S-matrix theory two points of view about the status of the analyticity postulate emerged.

(1) Chew emphasized the purely mathematical aspect. In his 1966 book The Analytic S-matrix he writes "In a deep sense physics is based on analytic functions. It is pointless to seek a logical origin for this circumstance. Physical theory cannot be based on logic; it is always a matter of guesswork based on observation of nature. One cannot, for example, argue that it is logical for classical mechanics to be expressible through second order differential equations. This simply is the scheme that works"

In his 1962 review Chew writes "the fundamental principle ... is of maximum smoothness ... and a natural mathematical definition of smoothness lies in the concept of analyticity".

* [illegible]'s Iagolnitzer & Stapp are concerned to link normal analytic structure in the physical region with their principle of macrocausality — this may be deducible from analyticity alone or may be an extra requirement. At all events a principle of maximal analyticity is required to extend the primitive domain of analyticity secured by the macrocausality principle.

note also work of Zeeman (1964) who uses causality to derive the Lorentz group, but this is sufficient, not necessary condition so macrocausality ~~to~~ could well be consistent with Lorentz group inputs of Zeeman's work.

thus Chew stresses on over what is conceived essentially with the "Jaffles structure"
This is what Mandelstam effectively criticises in his 1962 review.

(2) Stapp has tried to link analyticity with a macrocausality principle — causality may be allowed to fail over short space-time intervals — Indeed he claims microcausality may actually be inconsistent with axiomatic S-matrix theory (1962) (Iagolnitzer and Stapp (1969)) *
derives macrocausality & analytic structure in physical region
Stapp regards S-Matrix theory as expressing a "pragmatic attitude towards philosophy of QM" and does not appear entirely to share Chew's view of the primacy of purely mathematical considerations.

7. The Chew-Mandelstam Heuristic Stratagem

Effectively ~~what~~ the route to analytic S-Matrix theory is as follows:
(a) Mandelstam derives a property (of analyticity) by considering an approximation model, of field theory (i.e. 4th order perturbation theory)

(b) Mandelstam conjectures this property may be true of complete theory

(c) Chew now reinstates this conjecture as being "model-independent" by fiat and takes it as an ~~axiom~~ axiom for a new theory which may or may not be equivalent to (i.e. a reformulation of) the old theory.

We represent this sequence schematically in the following way.

original theory approximation

$T + A \longrightarrow T_1 \longrightarrow P$ (explains C)

approximate theory (model) property of the model

$\longrightarrow T'(P)$

new theory with
P incorporated as an axiom.

So in our case
- $T$ is R.Q.F.T.
- $A$ is Feynman perturbation approach
- $T_1$ is a class of Feynman diagrams
- $P$ is analytic property of these diagrams.
- $C$ is dispersion relations connecting observable quantities which are verified experimentally
- $T'(P)$ is axiomatic analytic S-matrix theory.

* The source of the C.D.D. ambiguity is that the Mandelstam conjecture tells us nothing about singularity at $\infty$ in complex plane — i.e. we may need subtractions which introduce arbitrary subtraction constants into Dispersion relations. Removal of C.D.D. poles is equivalent to an assumption about amplitude behaviour viz vanishing of all phase shifts at high energy (cf. Levinson's theorem)

Another example of this heuristic strategy would be Gell-Mann's current algebra (1962) in which commutation properties of currents are derived from a particular model — effectively the quark model — are made model-independent by fiat and taking as the starting point for constraining all subsequent theories.

## 8. C.D.D. ambiguities

The Mandelstam representation led to the possibility of formulating partial wave dispersion relations. But it turned out that the resulting integral equations did not have unique solutions — in particular an arbitrary number of poles could be introduced into the solution and the equations would still be satisfied. This ambiguity was already known from the study of the Chew–Low model and is known as the Castillejo–Dalitz–Dyson ambiguity (1956) (abbreviated C.D.D.). * This ambiguity was limited to partial waves $l=0$ or $1$ (by Froissart bound), and the possibility of removing the

* The reason why the Regge poles are ~~compos~~ regarded as composite, i.e. dynamical in origin is that the location of the poles depends on the strength of the interaction (which controls the shape of the Regge Trajectories in the J-plane)

The C.D.D. poles are fixed independent of the strength of the interaction.

ambiguity led to the idea of a bootstrap theory of the hadrons.

## 9. The Chew-Frautschi Bootstrap

Chew & Frautschi (1961) sought to eliminate the C.D.D. ambiguity by using Regge's ideas about analyticity in angular momentum. They introduced a principle of maximal analyticity of the second kind which says that all particle poles are Regge poles i.e. determined by unique analytic continuation in J from the unambiguous high angular momentum poles. C.F. interpret this situation as a nuclear democracy, the arbitrary C.D.D. ambiguities having been eliminated. *

There still remained the question of whether all arbitrary parameters could be eliminated from the theory. Chew & Frautschi (1962) also postulated a principle of maximum strength to fix the overall strength of

interactions they may in fact not be required as separate assumptions, but appear to be satisfied "experimentally".
The final objective of the bootstrap is to have no arbitrary parameters except a dimensional constant to fix the scale of the hadron masses.

In general there are now 3 possibilities

(1) there are several sets of particles that satisfy the bootstrap

(2) there is no set of particles that satisfy the bootstrap

(3) there is a unique set of particles that satisfy the bootstrap and these are the particles observed in nature.

The last possibility ~~was the~~ is what we may call the <u>Chew-Frautschi hypothesis</u>

## Examples of partial bootstraps

(1) $\rho = \pi\pi(\rho)$

Two pions "exchange" a $\rho$ which produces an interaction which leads the $\pi$'s to form the $\rho$

cf Chew & Mandelstam, Chew & Frautschi, Zachariasen & Zemach

(2) $N = \pi N(N)$

$\pi = N N(\pi)$

In this model $\pi$ & $N$ could both be bootstrapped

(3) The reciprocal bootstrap

$N = \pi N(N^*)$

$N^* = \pi N(N)$

cf Chew, Abers & Zemach

More generally we should write

$\{N, N^*\} = \pi N(N, N^*)$

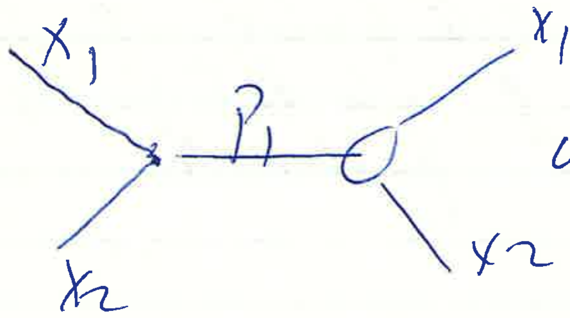
Results of partial bootstrap schemes are pretty rough, out from experimental values of masses & widths by factors of 2 or 3.

In general we write

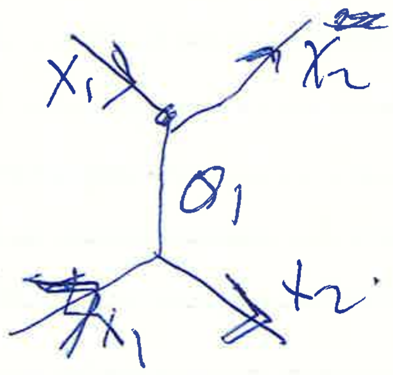
$$\{P_1 P_2 \ldots\} = X_1 X_2 \ldots (Q_1 Q_2 \ldots)$$

Composites  constituents  "exchange" particles.

<u>Example</u>


is bound pole




Some pole P may appear in many different reactions so we can write

$$P = X_1 + X_2 \text{ or } X_1 + X_2 + X_3 \text{ etc}$$

The point about the bootstrap is that the sets $\{P\}$ $\{X\}$ and $\{Q\}$ are all the ~~same~~ set the unique set of actually existing hadrons.

* So $X + A = (B + C) + X$

is not to be interpreted as A being broken up into B & C by collision with particle X.

This leads to some apparent anomalies
e.g. $A = B + C$
and $B = A + C$.

So A is part of B and B is part of A

or $A = A + B$, so A is part of itself.

This merely shows the inadequacy of a simple containment model

In fact it is best to think of energy in the incident system as being used to "create" the outgoing particles in a reaction

In this sense there is no anomaly in above examples when they are interpreted in terms of "creating" the emerging particles instead of "releasing" what is already there. *

Comparison of Neotrope philosophy with other philosophers

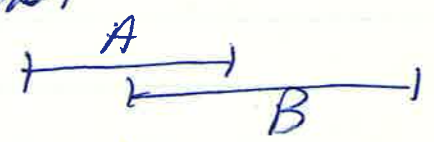
(1) Leibniz clear strongest analogy of Principle of Sufficient reason. (1968)
"Nature is as it is because this is the only possible world consistent with itself"
Two views of Leibniz
(a) Many possible worlds – existing world is chosen by God as best possible

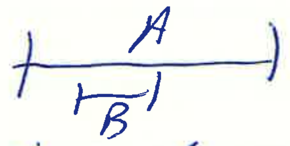
used
(2) Even existence is internally determined
( cf Russell's interpretation of Leibniz's secret philosophy.

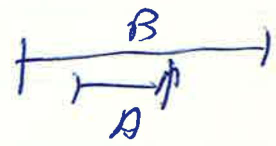
Clearly also borrows from Leibniz the idea that nothing in Nature is arbitrary.

(3) Anaxagoras claims everything substance contains every other substance.

But A's ~~Holes~~ view is essentially a containment model.
Contradictions are avoided by formulation seeds contain portions of all seeds in place of seeds contain all seeds.
e.g. 2 sets A & B may each include subsets of each other




But if 

then 

is impossible.

Also ~~the~~ for Anaxagoras the substances are immutable — i.e. it is a form of atomism

The Leibniz is much closer to the view that substances can change.

than fellow as e.g. Thales, Anaximenes, Anaximander and in particular Heraclitus.

Anaxagoras is much closer to Empedocles, Leucippus & Democritus all of whom derive from the Parmenidean reaction to Heraclitus.

(3) Analogies with Eastern philosophers such as Hinduism & Buddhism has been stressed by Capra in his book The Tao of Physics

(4) Analogy with Whitehead has been stressed by Stapp (1971). The link is a web philosophy of interlocking processes – refer to Whitehead's 'Process and Reality'.

Shortcomings of the bootstrap

1.) It makes life very difficult for physicists

2.) Partial bootstraps may be impossible to satisfy — we may have to include all particles at once

3.) Ideas may be untestable, and so unscientific due to enormous mathematical complexity of a full bootstrap.

4.) Bootstrap does not include the electron & photon — Chew claims these particles are associated with process of measurement, so should be treated differently.
But what about the other leptons — the muon & the neutrino?

5.) Chew makes some wild metaphysical claims. He says a complete bootstrap would demand for self-consistency "confronting the elusive concept of measurement and possibly even of consciousness".

* Kogut & Susskind (1973)

"It seems to us very unlikely that partons are really point particles . . . a parton will be resolved even further into yet another hierarchy of ~~components~~ constituents".

3 # The Ultimate Nature of Matter

## 1. The Bootstrap Picture

Formally we write
$$X_1 = \begin{cases} X_1 \\ X_a + X_u \\ X_a + X_u + X_c \\ \vdots \end{cases}$$
$$X_2 = \begin{cases} X_2 \\ X_a + X_u \\ X_a + X_2 + X_e \\ \vdots \end{cases}$$

so that each particle $X_1 X_2 \ldots$ may be represented as composed of other particles in many different ways corresponding to all the competing reaction channels to which the particle is linked.

## 2. Thalean Fundamentalism

The bootstrap equations have a soln in form

$$X_1 = \phi_1 + \phi_2 + \cancel{\phi_3} \ldots$$
$$X_2 = \phi_1 + \phi_2 + \phi_3 \ldots$$
$$\vdots$$

in terms of a set of fundamental objects $\phi_1 \phi_2 \ldots$ e.g. hadrons are explained in terms of quarks at a "deeper" level of structure.

These equations may be understood in simple containment sense, hadronic reactions being interpreted in terms of ~~rexchange~~ rearrangement of "unchanging" quarks. This could lead to an infinite regress. *

3. Anaximandrean Fundamentalism

Fundamental object is something different from ordinary matter (cf the Apeiron [apeiron] or Undifferentiated being of Anaximander)

Schematically we could write for particles as

$$X_1 = X_1(F)$$
$$X_2 = X_2(F)$$

i.e. all particles are explained as "excitations" of a single principle, the unified field F.

Matthews (1971) in his book The Nuclear Apple gives the analogy of particles as knots on a string.

— cf Heisenberg's Unified Field Theory.

4. Mathematical Atomism

Analogy between Plato's theory in Timaeus of building regular solids from two sorts of triangle and the SU(3) symmetry patterns being built up out of simple "triangular" operations

4. Conclusions

1. The Structure of Scientific Theories

~~1.~~ Our methodology of heuristics comprises features from several sources.

A From Kuhn it accepts paradigm shifts but only on the grand scale.

B From Lakatos it accepts hard-core & protective belt.

C From Popper it rejects normal science and also the working of the positive heuristic as "dull" science.

D From Toulmin it borrows the concept of a unit of variation in an evolutionary approach to the growth of science

~~E~~ E The unit of variation is identified with a micro research programme.

2. Correspondence Relations

~~3.~~ The relation between successive research programmes (micro & not) within a paradigm is one of Correspondence, which emphasizes the Conservative element in new theories which borrow structure from old theories. This feature of Correspondence has been analysed under the notion of a directional shift followed by a 'sketching' in the direction indicated

by a polarizing phenomenon or by a polarizing property of some model of the old theory

## 3. The Role of Surplus Structure

3. The role of surplus structure has been emphasized and one way of reformulating and stretching a theory is by altering the surplus structure of some extension of a mathematical model of the old theory. This emphasis on purely mathematical considerations is a feature of modern theoretical physics that is also apparent in the work of Einstein and Dirac.

## 4. The Floating Model

We have stressed the importance of the computation gap. If an approximate calculation disagrees with experiment we do not know whether to direct the modus tollens at the original theory or at the approximation.

In the case of atomic or molecular physics we have some confidence in the underlying theory because there are simple soluble problems such as the hydrogen atom or molecule which can be solved very accurately so these predictions really do test the theory not the theory + approximations. We can now argue that if in a more complicated problem (in nuclear physics or chemistry for example) a scheme of approximation

give results in agreement with experiment then we may believe this approximation has picked out the essential relevant features of some complicated dynamical situation i.e. we can justify the approximation a posteriori in virtue of its success.

But in particle physics ~~there are no~~ of strong interactions there are no simple isolatable problems — we have seen how solution of a one problem always involves simultaneous consideration of many other problems in the spirit of the bootstrap.

Hence our approximation models are not anchored to any secure underlying theory — in this sense they may be said to float.

Post for a stronger sense of floating model is that approximation models may also be used which do not agree with experiment — they float at both ends (theoretical and observational).

But such models are only defensible if they are not allied with a argument that the mismatch between the model and the experimental results is describable (theoretically) in a simple way. e.g. in Elliott's SU(3) model in Nuclear physics the broken symmetry is represented by a quadrupole interaction which does not mix different irreducible representations of the underlying symmetry of the harmonic oscillator potential (it is a function of the generators of the SU(3) symmetry)

5. The failure of the Scientific Method



8. In a sense the bootstrap philosophy as carried would point to a fundamental impasse for scientific method as we know it. The success of scientific method depends on the possibility of being able to isolate simple phenomena and of being able to disregard the enormous complexity of every real-life situation. The bootstrap philosophy would tell us that in the realm of hadron dynamics the approach is no longer possible. As a parallel one can think what celestial mechanics would be like if the planetary system was not susceptible to perturbation theory.

The bootstrap philosophy is essentially one of despair and frustration although Chew himself sees things just the other way around (1970)

"I would find it a crushing disappointment if in 1980 all of hadron physics could be explained in terms of a few arbitrary entities — we should then be in essentially the same posture as in 1930 -- to have learned so little in half a century would to me be the ultimate frustration".

I hope what I have said has been sufficient to substantiate ~~so~~ the claim I made in my opening remarks viz. that particle physics has exposed or at any rate emphasised ~~high lighted~~ some methodological problems which deserve the attention of philosophers of science.

---